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SHOCK AND EXPANSION WAVES IN TRANSONIC FLOW

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The present article is concerned with the propagation of a shock wave and a simple expansion wave in transonic flow. Approximate relations are obtained for the flow parameters, and the resulting asymptotic dependences are analyzed as the small parameters of transonic theory tend to zero. The derived equaitons are used to show that a universal relation of the kind that exists in the linear theory of supersonic flow between the optimum permeability coefficient and the freestream Mach number M does not exist independently of the flow-obstructing body at $M \gtrsim 1$ for the Darcy condition customarily used in the theory of linear induction of pipe walls.

Nikol'skii [1] has succeeded in obtaining a universal relation for the optimum permeability coefficient of a perforated wall in the case of supersonic pipe flow (the influence of the wall on the flow in the pipe is assumed to be completely eliminated), satisfying the Darcy condition $v/u + R = 0$ (u and v are the horizontal and vertical components of the perturbed velocity, and R is the perforation ratio). Assuming small deviations of the velocity from the freestream velocity, Nikol'skii showed that the relation $v/u = -\sqrt{M_1^2 - 1}$ holds in an unbounded flow both in the shock wave and in the expansion wave generated by an obstructing body and does not depend on the parameters of the body (M_1 is the freestream Mach number of the supersonic flow). This relation is proposed in [1] as the condition for obtaining noninductive flow in a supersonic pipe, where it is required that the permeability coefficient of the walls satisfy the equation $R_{opt} = \sqrt{M_1^2 - 1}$.

Here we investigate the flow properties in a shock wave and in an expansion wave when $M \approx 1$. We show that a unique functional relation for the optimum permeability coefficient of the wall no longer exists for near-sonic supersonic flow, and instead it varies along the length of the pipe wall in each flow situation and differs for each experiment.

1. We consider the exact equations for an oblique compression shock [2, 3] (Fig. 1)

$$\frac{p_2}{p_1} - 1 = \frac{2\gamma}{\gamma+1}\left(M_1^2 \sin^2\beta - 1\right); \tag{1.1}$$

$$\operatorname{tg}\theta = \frac{\sin^2\beta - 1/M_1^2}{(\gamma+1)/2 - \sin^2\beta + 1/M_1^2}\operatorname{ctg}\beta; \tag{1.2}$$

$$M_2^2 \sin^2(\beta - \theta) = \frac{1 + [(\gamma-1)/2]\, M_1^2 \sin^2\beta}{\gamma M_1^2 \sin^2\beta - (\gamma-1)/2}. \tag{1.3}$$

Here p is the pressure, θ and β are the flow turning angle and the angle of inclination of the shock front, both measured relative to the x axis, β_1 is the supplementary angle measured from the y axis, the subscripts 1 and 2 refer to the state of the flow before and after the shock front, and γ is the adiabatic exponent.

Transonic flow is known to be characterized by two small parameters, whose interrelationship is dictated by the particular transonic regime characterized by the similarity parameter. The parameters $M_1^2 - 1$ and $M_2^2 - 1$ are convenient choices for the analysis of flow around a compression shock. The angles $\theta \ll 1$ and $\beta_1 \ll 1$ are also small in this case and can

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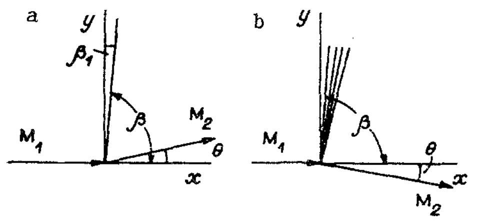

Fig. 1

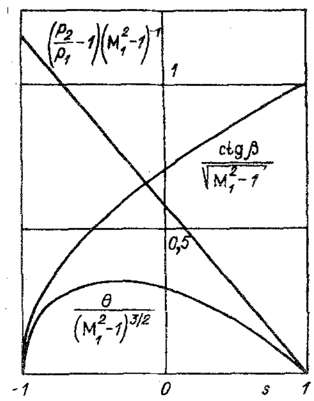

Fig. 2

be determined once the first two parameters have been specified. We assume that $M_1 > 1$ and we do not impose any condition on the sign of $M_2 - 1$, so that the flow can also be subsonic near the sonic point.

Expanding Eqs. (1.1)-(1.3) in the small parameters and keeping only the terms needed, we obtain the following equations in the principal approximation:

$$\frac{p_2}{p_1} - 1 = \frac{\gamma}{\gamma+1}(1-s)(M_1^2 - 1); \tag{1.4}$$

$$\theta = \frac{1}{\sqrt{2}(\gamma+1)}(1-s)(1+s)^{1/2}(M_1^2-1)^{3/2}; \tag{1.5}$$

$$\operatorname{ctg}\beta \approx \beta_1 = \frac{1}{\sqrt{2}}(1+s)^{1/2}(M_1^2-1)^{1/2}; \tag{1.6}$$

$$\sin\beta = 1 - \frac{1}{4}(1+s)(M_1^2-1), \tag{1.7}$$

where $s = (M_2^2 - 1)/(M_1^2 - 1)$. Taking into account the relations for u and v (perturbed velocity components) $u = -(1/\gamma)((p_2/p_1) - 1)$ and $v = \theta$, which are valid in the first approximation on the basis of the theory of small perturbations, we obtain

$$\frac{v}{u} = -\frac{1}{\sqrt{2}}(1+s)^{1/2}(M_1^2-1)^{1/2}. \tag{1.8}$$

The small quantities are independent, so that the parameter s can be chosen arbitrarily in the expressions preceding the factor $(M_1^2 - 1)$, but the range of variation of s is limited by Eq. (1,6): $-1 \leqslant s \leqslant 1$.

Thus, the expressions for the coefficients of $(M_1^2 - 1)$ have the distinctive feature that a double limit does not exist as the two parameters tend independently to zero. By specifying different values of s, we obtain different coefficients. These dependences are shown in Fig. 2.

We now consider some typical examples. Let the passage to the limit be such that $M_2 = 1$ and $s = 0$. The equations for the pressure, the turning angle, and the angle of inclination of the shock front then have the form

$$\frac{p_2}{p_1} - 1 = \frac{\gamma}{\gamma+1}(M_1^2-1), \quad \theta = \frac{1}{\sqrt{2}(\gamma+1)}(M_1^2-1)^{3/2}, \quad \operatorname{ctg}\beta = \frac{1}{\sqrt{2}}(M_1^2-1)^{1/2}.$$

On the other hand, if the passage to the limit corresponds to a normal shock (for which the flow turning angle is equal to zero, and $s = -1$), we have

$$\frac{p_2}{p_1} - 1 = \frac{2\gamma}{\gamma+1}(M_1^2-1), \quad \theta = 0, \quad \operatorname{ctg}\beta = 0,$$

where the velocity at the shock front is subsonic.

All other transitions are legitimate. In particular, for $s = 1$ we obtain a Mach line, which corresponds to the well-known regime from the theory of characteristics for the equations of supersonic gas dynamics:

$$\operatorname{ctg}\beta = \sqrt{M_1^2 - 1}.$$

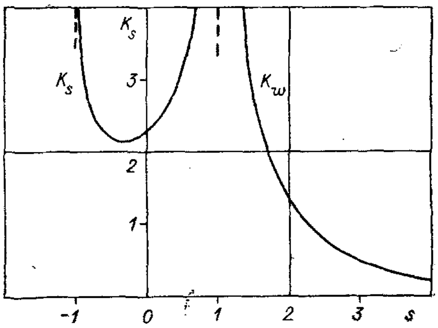

Fig. 3

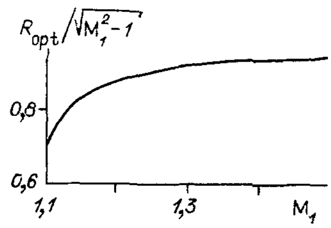

Fig. 4

Each regime corresponds to a value of the Kármán similarity parameter

$$K_s = (M_1^2 - 1)/\theta^{2/3} = 2^{1/3}(\gamma + 1)^{2/3}(1 + s)^{-1/3}(1 - s)^{-2/3}.$$

We note that K_s has a lower bound ($s = -1/3$) in the vicinity of the shock wave in transonic flow: $3\cdot 4^{-2/3}(\gamma + 1)^{2/3} \leqslant K_s < \infty$.

The function $K_s(s)$ is plotted in Fig. 3. The minimum value of K_s is attained in the subsonic region after the shock. The transition through the postshock sonic point ($M_2 = 1$) takes place at $s = 0$. The fact that K_s has a lower bound (a regime with $K_s = 0$ is impossible in the vicinity of the shock) must be taken into account in the study of flows with weak shocks in the approximation of the Tricomi equation, where the transonic similarity parameter is assumed to be equal to zero for the entire region.

2. We now consider flow in a Prandtl–Meyer expansion wave, in which case the flow velocities before ($M_1 \geqslant 1$) and after ($M_2 > 1$) the wave are close to the sound velocity (Fig. 1b). Invoking the exact equations for an expansion wve [2, 3] in terms of the flow turning angle and the pressure and expanding it in the small parameters, we obtain

$$\theta = \frac{2}{3(\gamma+1)}[(M_2^2 - 1)^{3/2} - (M_1^2 - 1)^{3/2}]; \tag{2.1}$$

$$\frac{p_2}{p_1} - 1 = \frac{\gamma}{\gamma+1}[(M_1^2 - 1) - (M_2^2 - 1)]. \tag{2.2}$$

Introducing the parameter $s = (M_2^2 - 1)/(M_1^2 - 1)$ as before, we have

$$\theta = \frac{2}{3(\gamma+1)}(s^{3/2} - 1)(M_1^2 - 1)^{3/2}; \tag{2.3}$$

$$\frac{p_2}{p_1} - 1 = \frac{\gamma}{\gamma+1}(1 - s)(M_1^2 - 1),\ s \geqslant 1; \tag{2.4}$$

$$\frac{v}{u} = \frac{2}{3}\frac{s^{2/3} - 1}{s - 1}(M_1^2 - 1)^{1/2}. \tag{2.5}$$

Again, the solution corresponding to an expansion wave is not unique for the coefficients in Eqs. (2.3) and (2.4); different coefficients are obtained in the limit $(M_1^2 - 1) \to 0$, depending on the value of s. For example, when sonic flow $M_1 = 1$ expands to the value M_2 [corresponding to $s = \infty$, so that the limit must be calculated from Eqs. (2.1) and (2.2)], we find

$$\theta = \frac{2}{3(\gamma+1)}(M_2^2 - 1)^{3/2},\quad \frac{p_2}{p_1} - 1 = -\frac{\gamma}{\gamma+1}(M_2^2 - 1).$$

This regime corresponds to the transonic parameter $K_w = (M_1^2 - 1)/\theta^{2/3} = 0$. It is evident from Eqs. (2.3) and (2.4) that other values of K_w correspond to different values of the coefficients in the equations for the flow turning angle and the pressure; K_w is related to s by the equation $K_w = (3/2)^{2/3}(\gamma + 1)^{2/3}(s^{3/2} - 1)^{-1}$. This dependence is shown in Fig. 3.

3. The indicated multiple-valued ambiguity of the resulting dependences in the transonic velocity range is a fundamental departure from the familiar, perfectly single-valued, supersonic flow equations, where the perturbations are small in comparison with the parameter $M_1^2 - 1$. i.e., $|M_2 - M_1| \ll M_1 - 1$.

For weak compression shocks this condition is equivalent to the assumption that the difference between the angles of inclination of the shock front and the characteristic is much smaller than the angle of deviation β_1 of the compression shock from the vertical (see

Fig. 1a). This condition yields an additional relationship between the two parameters (M_1 and M_2), closes the system (1.1)-(1.3), and makes the dependence of the coefficients on M_1 single-valued for all flow deviation angles as long as they are not too large. This is why the dependence of the optimum permeability coefficient of the wall on M in supersonic flow is universal [1].

For flow that is supersonic but in the transonic regime, a unique dependence no longer exists for the optimum permeability coefficient; it varies along the length of the pipe wall in each case and differs for each experiment. To illustrate this fact, we give the example of the calculation of a gas flow around a slender wedge at zero angle of attack with a velocity slightly greater than the sonic velocity. The half-angle of the wedge is $\theta = 1°22'$.

Figure 4 shows how R_{opt}, normalized to $\sqrt{M_1^2 - 1}$, varies as a function of M_1 in the region above the wedge behind the compression shock. The calculations have been carried out according to the exact gasdynamic equations and tables [4].

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AN EXACT SOLUTION FOR THE END EFFECT OF A WING OF FINITE SIZE IN SUPERSONIC FLOW

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The problem of supersonic flow over a thin wing of finite size, examined in a linear approximation, reduces to solving the wave equation for the velocity potential. The condition that the flow does not penetrate the wing surface is then carried to the base plane, and in the remainder of this plane (outside the projection of the wing) certain conditions are imposed on the gasdynamic parameters of the flow. The solution of the problem is given in [1] when the velocity potential is determined via the normal derivative in the base plane Φ'_η, and outside the wing projection we have the condition that the potential goes to zero. The gasdynamic flow parameters (pressure, downwash outside the wing) obtained from this solution take on physically invalid infinite values in the vicinity of the subsonic leading edge. Expressions are given in [2] for the velocity potential and its derivatives in terms of the first and second derivatives of the potential in the base plane, which allows one to apply additional boundary conditions and obtain a solution of the flow problem in which the gasdynamic flow parameters are in a class of bounded functions.

This paper derives formulas for calculating the gasdynamic flow parameters in the case when the velocity potential is determined [2] via the first derivative Φ'_η and the second derivative $\Phi''_{\eta\xi}$ (the surface curvature in the incident stream direction) in the base plane, and in the part of the base plane outside the wing projection the condition of continuity of the derivative Φ'_ξ (pressure) is applied.

1. The velocity potential at the point M(x, y, z) lying in the perturbed region above the wing is found via the normal derivative Φ'_η in the base plane $\eta = 0$ from the formula [1]

$$\Phi = -\frac{1}{\pi} \iint\limits_{s+\sigma} \Phi'_\eta(\xi, \zeta)\, \varphi\, d\xi\, d\zeta, \tag{1.1}$$

where $\varphi = r^{-1}$; $r = \sqrt{(x-\xi)^2 - (z-\zeta)^2 - y^2}$; $(s+\sigma)$ is the region of dependence of the point M in the plane $\eta = 0$ (Fig. 1). Part of the region of dependence $s(COO_1D_1M_0C)$ coincides with the

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